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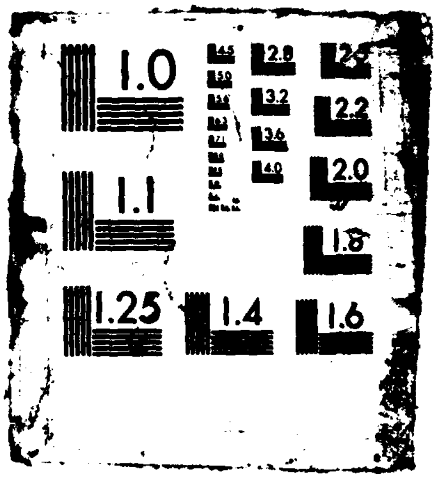
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CONVERGENCE OF RELAXATION ALGORITHMS BY AVERAGING

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ABSTRACT

Adaptive relaxation algorithms use anti-jamming schemes that require either a large amount of computation or a large amount of memory. ~~In~~ this paper ~~we~~ presents a non-adaptive approach that possesses substantial computational and memory advantages over the adaptive schemes. The approach uses averaging and may be applied whenever the relaxation algorithm's point-to-set maps satisfy appropriate assumptions.

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I. INTRODUCTION

There exist many situations ([1]-[3], [5], [8], [10], [12]-[16]) in which the solution set $D(P)$ (assumed to be in E^n for simplicity's sake) of a problem P may be expressed as the intersection of the solution sets $D(P_j)$ of a finite number of problems P_j, $j = 1$, $2,..p$, that is $D(P) = D(P_1) \cap D(P_2) \cap \cdots \cap D(P_p)$. In that case, one may try to find a set T, point-to-set maps $A_j(.)$, $j = 1, 2,..., p$, and a surrogate cost $v(.)$ so that the following hypotheses are satisfied.

Hypothesis 1:

(i) T is a closed subset of E^n.

(ii) $v(.)$ is from T into E.

For $j = 1, 2,..., p$:

(iii) $A_j(.)$ is from T into all the non-empty subsets of T,

(iv) $D(P_j) = \{z \in T \mid z \in A_j(z)\}$.

Hypothesis 2: For $j = 1, 2,..., p$, if a point z is in T but not in $A_j(z)$, scalars $\epsilon_j(z) > 0$, $\delta_j(z) > 0$ and $\lambda_j(z)$ exist such that $v(y') \leq v(z') - \delta_j(z)$, and $\lambda_j(z) \leq v(z')$ for every y' in $A_j(z')$ and for every z' in $B(z, \epsilon_j(z)) \cap T$.

Hypothesis 1 ensures that the relaxation algorithm given below is well defined and Hypothesis 2 is the usual monotonicity assumptions used to obtain asymptotic stability in the large [4], [6], [7], [11], [17], [18].

Starting at some point z_1 in T, one generates a sequence $\{z_i\}$ by using one of the maps $A_j(.)$ at iteration i.

Algorithm 1: Let z_1 in T be given.

Step 0: Set $i = 1$.

Step 1: Find a point x_i in $A_{m(i)}(z_i)$, $1 \leq m(i) \leq p$.

Step 2: If $v(x_i) \geq v(z_i)$, let $z_{i+1} = z_i$, let $i = i+1$, and go to Step 1; else, let $z_{i+1} = x_i$, let $i = i+1$ and go to Step 1.

If Algorithm 1 uses point-to-point maps $a_j(.)$, that is, if $A_j(z)$ contains one and only one point $a_j(z)$ for every z in T and $j = 1, 2, \ldots, p$, if those maps are continuous on T, and if Hypotheses 1 and 2 are satisfied, then every cluster point of every sequence generated by Algorithm 1 is in $D(P)$, and furthermore, if $D(P)$ contains at most a countable number of points, every bounded sequence generated by Algorithm 1 converges to a point in $D(P)$, provided that $\{m(i)\}$ satisfies the following hypothesis proposed by Fiorot and Huard [2], [3, p. 76, Hypothesis H5]:

Hypothesis 3: An integer r exists so that to every $i = 1, 2, \ldots$ and j in the interval $[1,p]$ correspond at least one index k in $[i, i+r]$ such that $m(k) = j$.

It is not difficult to find maps $m(.)$ that satisfy Hypothesis 3: a cyclic approach corresponds to letting $m(1) = 1, m(2) = 2, \ldots, m(p) = p, m(p+1) = p+1, \ldots$, etc, and Aitken double sweep choice [4, p. 158] corresponds to letting $m(1) = 1, m(2) = 2, \ldots, m(p) = p, m(p+1) = p-1, m(p+2) = p-2, \ldots$, etc. For examples of the approach used in the context of unconstrained minimization, see Luenberger [4, pp. 158-159].

Unfortunately, when Algorithm 1 uses point-to-point maps that are not continuous or point-to-set maps, its asymptotic properties may not be related to $D(P)$ ([3], [12]). In such cases one may use one of the two adaptive schemes given in [7] for determining the quantity $m(i)$ at iteration i. Both schemes have drawbacks: either a large amount of computation or a large amount of memory is required. To alleviate those difficulties, we propose an algorithm based on averaging that uses a sequence $\{m(i)\}$ that may be chosen non-adaptively.

II. A NON-ADAPTIVE AVERAGING SCHEME

The asymptotic properties of the averaging algorithm are related to $D(P)$ when, in addition to Hypothesis 1, the following hypothesis is satisfied.

Hypothesis 4:

(i) The set T is convex.

(ii) The map $\nu(.)$ is upper-semi continuous on T with respect to T, that is, to every point z in T and $\delta > 0$ corresponds an $\varepsilon > 0$ such that $\nu(z') \leq \nu(z) + \delta$ for every z' in $B(z,\varepsilon) \cap T$.

For every index j in $\{1, 2,..., p\}$:

(iii) if a point z is in T but not in $A_j(z)$, scalars $\varepsilon_j(z) > 0$, $\delta_j(z) > 0$ and $\lambda_j(z)$ exist such that $\nu(z' + \mu(y' - z')) \leq \nu(z') - \mu\delta_j(z)$, and $\lambda_j(z) \leq \nu(z')$ for every μ in the interval $[0,1]$, for every y' in $A_j(z')$ and for every z' in $B(z, \varepsilon_j(z)) \cap T$,

(iv) if a point z is in $A_j(z)$, then to every scalar $\delta > 0$ corresponds a scalar $\varepsilon > 0$ such that $\nu(z) - \delta \leq \nu(y')$ for every y' in $A_j(z')$ and for every z' in $B(z,\varepsilon) \cap T$.

The averaging algorithm uses an initial guess of a solution z_1 in T and a sequence $\{m(i)\}$ that takes its values in the set $\{1, 2,..., p\}$.

Algorithm 2: Given z_1 in T and $\{m(i)\}$ in $\{1, 2,..., p\}$

Step 0: Set $i = 1$.

Step 1: Find a point y_i in $A_{m(i)}(z_i)$.

Step 2: If $\nu(y_i) \geq \nu(z_i)$, let $z_{i+1} = z_i$, let $i = i+1$ and go to Step 1; else go to Step 3.

Step 3: Let $\mu_i = \min\left\{1, \dfrac{\nu(z_i) - \nu(y_i)}{\|y_i - z_i\|}\right\}$.

Step 4: Let $z_{i+1} = z_i + \mu_i(y_i - z_i)$, let $i = i+1$ and go to Step 1.

Lemma 1: If Hypotheses 1 and 4 are satisfied, then Algorithm 2 is well defined, and whenever z_{i+1} and z_i are two consecutive points of a sequence generated by the algorithm such that $z_{i+1} \neq z_i$, then $v(z_{i+1}) < v(z_i)$, and $||z_{i+1}-z_i|| \leq v(z_i)-v(y_i)$.

Proof: Parts (ii) and (iii) of Hypothesis 1, Part (i) of Hypothesis 4, and the fact that if $v(y_i) < v(z_i)$, the quantity μ_i is in [0,1] immediately imply that the algorithm is well defined. If z_{i+1} and z_i are two consecutive points of a sequence generated by Algorithm 2, and if $z_{i+1} \neq z_i$, then it is clear that $v(y_i) < v(z_i)$, $\mu_i \neq 0$, $z_{i+1} = z_i + \mu_i(y_i - z_i)$ and from (iii) of Hypothesis 4, $v(z_{i+1}) < v(z_i)$. If $\mu_i < 1$, then $||z_{i+1}-z_i|| = v(z_i)-v(y_i)$, if $\mu_i = 1$, then $z_{i+1} = y_i$, $v(z_i)-v(y_i) \geq ||y_i - z_i||$, and thus, if $z_{i+1} \neq z_i$,

$$v(z_i)-v(y_i) \geq ||y_i-z_i|| = ||z_{i+1}-z_i||. \square$$

We now analyze the asymptotic properties of Algorithm 2.

Lemma 2: Suppose that Hypotheses 1 and 4 are satisfied and let $\{z_i\}$ be a sequence generated by Algorithm 2. If an infinite subset K of the integers exists so that (i) the subsequence $\{z_i\}_K$ converges to some point z_*, (ii) the subsequence $\{y_i\}_K$ is bounded and (iii) $m(i) = j$ for every index i in K, then z_* belongs to $A_j(z_*)$, the subsequence $\{ ||z_{i+1} - z_i|| \}_K$ converges to 0, and the subsequence $\{z_{i+1}\}_K$ converges to z_*.

Proof: The set T is closed, the sequence $\{z_i\}$ is in T, and thus z_* is in T. Assume that z_* does not belong to $A_j(z_*)$. Part (iii) of Hypothesis 4 and the fact that $m(i) = j$ for every index i in K imply that an index k and a scalar $\delta_j(z_*) > 0$ exist such that

$$v(z_i + \mu(y_i - z_i)) \leq v(z_i) - \mu\delta_j(z_*) \tag{3}$$

for every μ in [0,1] and for every $i \geq k$, i in K.

Using Eq. (3) with $\mu = 1$ yields

$$v(y_i) \leq v(z_i) - \delta_j(z_*). \tag{4}$$

Thus, for every $i \geq k$, i in K, $v(y_i) < v(z_i)$ and μ_i is computed in Step 3 of Algorithm 2. The subsequences $\{z_i\}_K$ and $\{y_i\}_K$ are bounded and thus a scalar $\Delta > 0$ exists so that $||y_i - z_i|| \leq \Delta$ for all i in K. Using Eq. (4) we may immediately conclude that for every i

$\geq k$, i in K

$$\frac{v(z_i)-v(y_i)}{\|y_i-z_i\|} \geq \frac{\delta_j(z_*)}{\Delta},$$

and

$$\mu_i \geq \mu_{\min} = \min\left[1, \frac{\delta_j(z_*)}{\Delta}\right].$$

It follows that for every $i \geq k$, i in K

$$v(z_i + \mu_i(y_i - z_i)) \leq v(z_i) - \mu_{\min}\delta_j(z_*)$$

and

$$v(z_{i+1}) \leq v(z_i) - \mu_{\min}\delta_j(z_*). \tag{5}$$

From Lemma 1 we know that $v(z_{i+1}) \leq v(z_i)$ for every index i and thus Eq. (5) implies that the sequence $\{v(z_i)\}$ is unbounded from below. This is not possible in view of Hypothesis 4, part (iii), and we must conclude that z_* belongs to $A_j(z_*)$. Suppose now that $\{\|z_{i+1}-z_i\|\}_K$ does not converge to 0, that is, suppose that an infinite subset M of K and a scalar $\Delta > 0$ exist so that $\|z_{i+1}-z_i\| \geq \Delta$ for every i in M. Lemma 1 implies that

$$v(z_i)-v(y_i) \geq \|z_{i+1}-z_i\|,$$

and

$$v(y_i) \leq v(z_i) - \Delta. \tag{6}$$

The point z_* is in $A_j(z_*)$, and part (iv) of Hypothesis 4 implies the existence of a scalar $\varepsilon > 0$ such that $v(z_*) - \frac{\Delta}{2} \leq v(y)$ for every y in $A_j(z)$ and for every z in $B(z_*,\varepsilon) \cap T$. The subsequence $\{z_i\}_M$ converges to z_*, the point y_i is in $A_j(z_i)$ for every index i in M, and therefore an integer m exists so that for every $i \geq m$, i in M, z_i is in $B(z_*,\varepsilon) \cap T$ and thus

$$\nu(z_*) - \frac{\Delta}{2} \leq \nu(y_i). \tag{7}$$

Using Eqs. (6) and (7), we obtain

$$\nu(z_*) + \frac{\Delta}{2} \leq \nu(z_i) \tag{8}$$

for every $i \geq m$, i in M. Eq. (8) contradicts part (ii) of Hypothesis 4 and we must conclude that $\{\|z_{i+1} - z_i\|\}_K$ converges to 0. The subsequence $\{z_i\}_K$ converges to z_* and therefore the subsequence $\{z_{i+1}\}_K$ converges to z_* also. □

Lemma 3: Suppose that Hypotheses 1 and 4 are satisfied and let $\{z_i\}$ be a sequence generated by Algorithm 2. If an infinite subset K of the integers exists so that the subsequence $\{z_i\}_K$ converges to some point z_* and the subsequence $\{y_i\}_K$ is bounded, then the subsequence $\{z_{i+1}\}_K$ also converges to z_*.

Proof: Let the sets $K(j)$, $j = 1, 2, \ldots, p$, J and L be defined as follows:

$$K(j) = \{i \in K \mid m(i) = j\}$$

$$J = \{j \mid K(j) \text{ contains infinitely many indices}\}$$

$$L = \bigcup_{j \in J} K(j)$$

It is clear that $\{z_i\}_{K(j)}$ converges to z_* for every j in J. Lemma 2 implies that $\{z_{i+1}\}_{K(j)}$ converges to z_* for every j in J, and thus $\{z_{i+1}\}_L$ converges to z_*. The definition of L implies that an integer k exists so that

$$\{i \in K \mid i \geq k\} = \{i \in L \mid i \geq k\}$$

and thus the subsequence $\{z_{i+1}\}_K$ converges to z_*. □

The sequence $\{m(i)\}$ cannot be arbitrary if the asymptotic properties of Algorithm 2 are to be related to $D(P)$. Known results ([2], [3]) indicate that $\{m(i)\}$ should not only take on each value j, $j = 1, 2, \ldots, p$ for infinitely many indices i, but that the "density" of occurrence of these values must not become vanishingly small, that is, the map $m(.)$ must satisfy Hypothesis 3.

Theorem 1: Assume that Hypotheses 1, 3 and 4 are satisfied, and let $\{z_i\}$ be a sequence generated by Algorithm 2. If the sequence $\{y_i\}$ is bounded, then $\{z_i\}$ is bounded, every cluster point of $\{z_i\}$ is in $D(P)$, and $\{z_i\}$ is asymptotically regular, that is, the sequence $\{ \|z_{i+1} - z_i\| \}$ converges to 0.

Proof: The definition of Algorithm 2 implies that z_{i+1} is in the convex closure of the set of points $z_1, y_1, y_2, \ldots, y_i$, and thus the boundedness of $\{y_i\}$ implies the boundedness of $\{z_i\}$. Let z_* be a cluster point of a sequence $\{z_i\}$ generated by Algorithm 2. An infinite subset K of the integers exists so that the subsequence $\{z_i\}_K$ converges to z_*. Hypothesis 3 implies that an integer r exists such that $\{1, 2, \ldots, p\} = \{m(i), m(i+1), \ldots, m(i+r)\}$ for every positive integer i. Lemma 3 implies that the subsequences $\{z_{i+1}\}_K, \{z_{i+2}\}_K, \ldots, \{z_{i+r}\}_K$ converge to z_*. Let the sets L and $K(j)$, $j = 1, 2, \ldots, p$ be defined as follows:

$$L = \{i \mid i \in K\} \cup \{i \mid i\text{-}1 \in K\} \cup \cdots \cup \{i \mid i\text{-}r \in K\},$$

$$K(j) = \{i \in L \mid m(i) = j\}.$$

The definition of L implies that if i is in K, then $i, i+1, i+2, \ldots,$ and $i+r$ are in L. Thus to every j in $\{1, 2, \ldots, p\}$ and i in K correspond an index $i(j)$ in L such that $i(j) \geq i$, $i(j) - i \leq r$, and $m(i(j)) = j$. It follows that $K(j)$ contains infinitely many indices for every j in $\{1, 2, \ldots, p\}$, and using Lemma 2, we may then conclude that z_* is in $A_j(z_*)$ for every j in $\{1, 2, \ldots, p\}$, and thus in $D(P)$. The proof that $\{z_i\}$ is asymptotically regular is similar to the proof that $\{ \|z_{i+1} - z_i\| \}_K$ converges to 0 in Lemma 2 and has been omitted. □

The cluster point set of an asymptotically regular and bounded sequence is not arbitrary: it contains either one point or an uncountable number of points. This result, shown by Ostrowski [9, p.173] may be used to show that sequences generated by Algorithm 2 converge.

Corollary 1: Assume that Hypotheses 1, 3 and 4 are satisfied. If $D(P)$ contains at most a countable number of points and if T is bounded, then every sequence $\{z_i\}$ generated by Algorithm 2 converges to a point in $D(P)$.

Note that it is possible to parametrize Algorithm 2 by a scalar $\alpha > 0$. Let μ_i in Step 3 of Algorithm 2 be defined by

$$\mu_i = \min\left\{1, \frac{\alpha(v(z_i)-v(y_i))}{\|y_i-z_i\|}\right\}.$$

Theorem 1 and Corollary 1 hold for every $\alpha > 0$, and therefore it is possible to control when the anti-jamming feature of Algorithm 2 takes effect through the choice of α.

III. CONCLUSION

Relaxation algorithms may be used whenever the problem to be solved exhibit the appropriate structure, but anti-jamming schemes may have to be present to insure the desired convergence properties. To illustrate that point, we consider now the coordinate descent method for unconstrained minimization.

Let $T = E^n$, and given a continuously differentiable map $v(.)$ from T into E, and n linearly independent vectors e_j, $j = 1, 2, \ldots, n$ in E^n, let

$$D(P) = \{z \mid \nabla v(z) = 0\}$$

and for $j = 1, 2, \ldots, n$, let

$$D(P_j) = \{z \mid \langle \nabla v(z), d_j \rangle = 0\}.$$

Thus, problem P consists in finding a point in $D(P)$ and it is clear that a point z is a solution of P if and only if it is a common solution of the problems P_j, $j = 1, 2, \ldots, n$, where each problem P_j has a solution set $D(P_j)$. For every $j = 1, 2, \ldots, n$, let $A_j(.)$ be the map from T into all the subsets of T defined by

$$A_j(z) = \{y = z + \lambda d_j \mid \langle \nabla v(y), d_j \rangle = 0\}.$$

Given z in T and j in $\{1,2,...,n\}$, it is possible to obtain points in $A_j(z)$ by minimizing $v(.)$ over the line passing through z with direction d_j. Coordinate descent methods for minimizing $v(.)$ consists in using Algorithm 1 with $p = n$ and a sequencing map $m(.)$ that satisfies Hypothesis 3. Such methods are useful when the structure of $v(.)$ is such that its minimization along the privileged directions d_j is easy. Although it is sometimes believed that such methods produce the desired results, the examples given in [3] and [12] show that jamming may occur, that is, the sequences $\{\nabla v(z_i)\}$ that correspond to the sequences $\{z_i\}$ generated by Algorithm 1 may be bounded away from 0. To prevent jamming, one may use either one of the two adaptive schemes given in [7] or the averaging scheme presented in this paper. The non-adaptive schemes have a costly overhead that results from having to determine $m(i)$ at every iteration i. The scheme (Algorithm 2) presented in this paper is non-adaptive: $\{m(i)\}$ is given and thus the overhead needed to implement anti-jamming consists only in the determination of the appropriate step length. The efficiency gained by using averaging is obtained at a price: the assumptions that insure the convergence of Algorithm 2 are stronger than the assumptions that insure the convergence of the adaptive algorithms given in [7]. Fortunately, Hypothesis 4 is satisfied in most cases and the averaging scheme is therefore applicable. For example, the reader may verify that the averaging scheme can be used to prevent jamming when the coordinate descent method is used to minimize the three functions proposed by Powell in [12].

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